***Section* 3.5 – The Ratio and Root Tests**

***Theorem* − The Ratio Test**

Let  be a series with positive terms and suppose that



Then

1. the series ***converges*** if *ρ* < 1,
2. the series ***diverges*** if *ρ* > 1, or *ρ* is infinite
3. the test is ***inconclusive*** if *ρ* = 1,

The value *ρ* doesn’t mean the sum of the series.

***Example***

Investigate the convergence of the series 

***Solution***















The series ***converges*** since *ρ* < 1.









***Example***

Investigate the convergence of the series 

***Solution***

















The series ***diverges*** since *ρ* > 1.

***Example***

Investigate the convergence of the series 

***Solution***















Because the limit is *ρ* = 1, we can’t decide from the *Ratio Test* whether the series converges.

However, since , then the series ***diverges***.

***Theorem* − The Root Test**

Let  be a series with  for , and suppose that



Then

1. the series ***converges*** if *ρ* < 1,
2. the series ***diverges*** if *ρ* > 1, or *ρ* is infinite
3. the test is ***inconclusive*** if *ρ* = 1,

***Example***

Determine if the series  converges or diverges using the Root Test

***Solution***













The series ***converges*** by the *Root Test*.

***Example***

Determine if the series  converges or diverges using the Root Test

***Solution***













The series ***diverges*** by the *Root Test*.

***Example***

Determine if the series  converges or diverges using the Root Test

***Solution***





The series ***converges*** by the *Root Test*.

***Exercises*** ***Section* 3.5 – The Ratio and Root Tests**

Use the ***Ratio Test*** to determine if the series converges or diverges.

|  |  |  |
| --- | --- | --- |
|  |  |  |

Use the ***Root Test*** to determine if the series converges or diverges.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
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|  | |  | |  | |

Use any method to determine if the series converges or diverges.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
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|  | |  | |  | |

1. Use the integral test to show that  converges. Show that the sum *s* of the series is less than 
2. Use the root test to show that  converges
3. Use the root test to test that  converges
4. Try to use the ratio test to determine whether  converges. What happen?

Now observe that 



Does the given series converge? Why or why not?

1. Suppose  and  for all *n*. Show that  diverges.



1. Working in the early 1600s, the mathematicians Wallis, Pascal, and Fermat were calculating the area of the region under the curve  between  and , where *p* is the positive integer. Using arguments that predated the Fundamental Theorem of Calculus, they were able to prove that



Use Riemann sums and integrals to verify this limit.

1. Complete the following steps to find the values of  for which the series  converges
2. Use the Ratio Test to show that  converges for .
3. Use Stirling’s formula,  for large *k*, to determine whether the series converges when .

